



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 42

Option Trading Strategies – 3

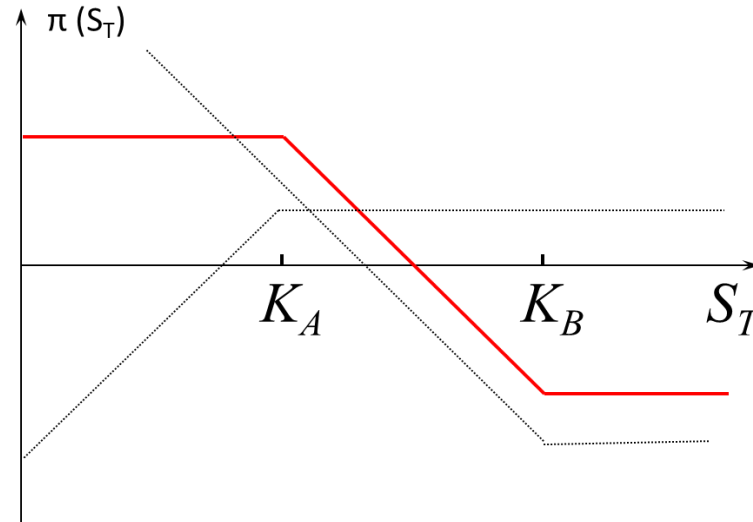
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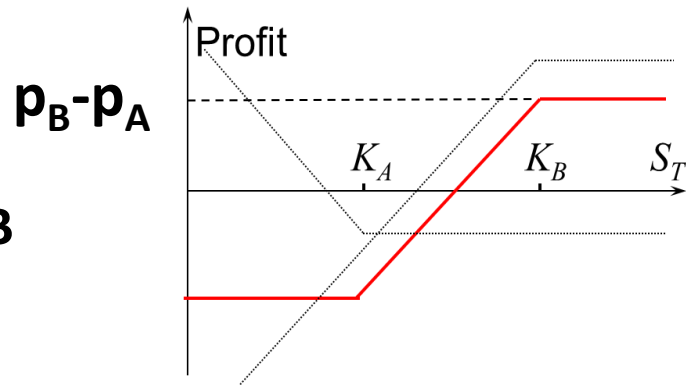
BEARISH CALL SPREAD

- SELL CALL A (K_A, T, c_A)
- BUY CALL B (K_B, T, c_B)
- $K_A < K_B$
- $c_A > c_B$



BULLISH & BEARISH PUT SPREADS

- BULLISH PUT SPREAD
- BUY PUT A (K_A, T, p_A)
- SELL PUT B (K_B, T, p_B), $K_A < K_B$
- BEARISH PUT SPREAD
- SELL PUT A (K_A, T, p_A)
- BUY PUT B (K_B, T, p_B), $K_A < K_B$



$$\pi_{\text{BULL PUT SPREAD}} = \pi_{\text{LONG PUT A}} + \pi_{\text{SHORT PUT B}}$$

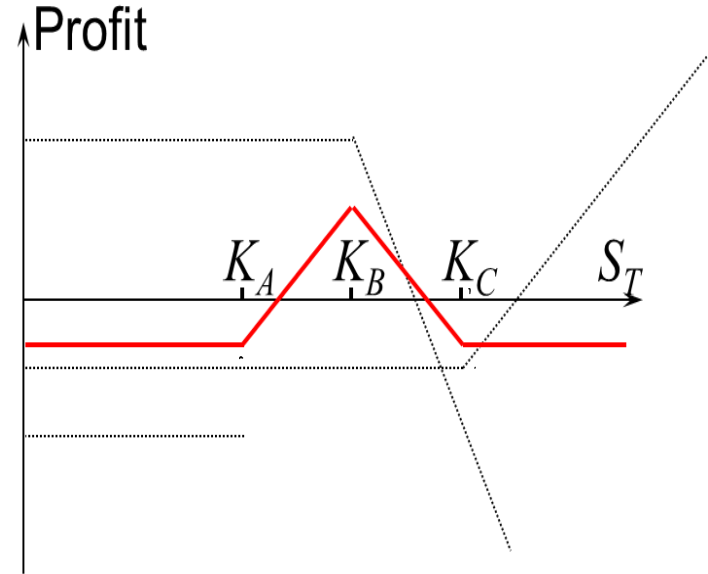
$$= \begin{cases} (\mathbf{K_A - S_T}) - (\mathbf{K_B - S_T}) + (\mathbf{p_B - p_A}) & \text{if } \mathbf{S_T < K_A < K_B} \\ -(\mathbf{K_B - S_T}) + (\mathbf{p_B - p_A}) & \text{if } \mathbf{K_A < S_T < K_B} \\ (\mathbf{p_B - p_A}) & \text{if } \mathbf{K_A < K_B < S_T} \end{cases}$$

$$= \begin{cases} -(\mathbf{K_B - K_A}) + (\mathbf{p_B - p_A}) (< 0) & \text{if } \mathbf{S_T < K_A < K_B} \\ (\mathbf{S_T - K_B}) + (\mathbf{p_B - p_A}) & \text{if } \mathbf{K_A < S_T < K_B} \\ (\mathbf{p_B - p_A}) (> 0 \text{ ALWAYS}) & \text{if } \mathbf{K_A < K_B < S_T} \end{cases}$$



BUTTERFLY CALL SPREAD

- BUY CALL A (K_A, T, c_A)
- BUY CALL C (K_C, T, c_C)
- SELL TWO CALLS B (K_B, T, c_B)
- $K_A < K_B < K_C$
- $K_C - K_B = K_B - K_A$ or
- $K_C + K_A = 2K_B$



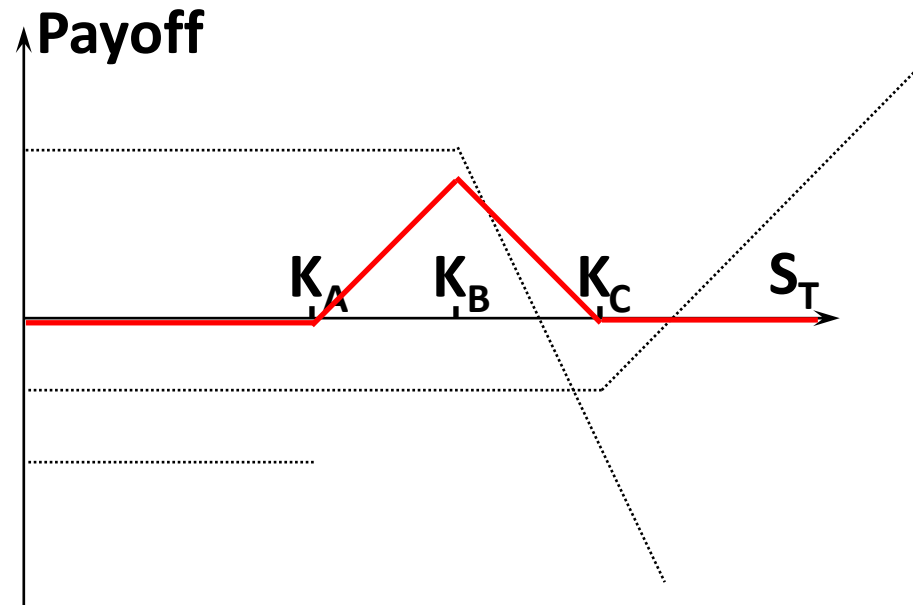
BUTTERFLY CALL SPREAD CONTD....

- We can divide the entire spectrum of stock price into four segments viz.
- $S_T < K_A < K_B < K_C$: ALL CALLS WILL LAPSE
- $K_A < S_T < K_B < K_C$: A WILL BE EXERCISED, B & C WILL LAPSE
- $K_A < K_B < S_T < K_C$: A & B WILL BE EXERCISED, C WILL LAPSE
- $K_A < K_B < K_C < S_T$: ALL CALLS A, B & C WILL BE EXERCISED



BUTTERFLY CALL SPREAD ($K_C + K_A = 2K_B$)

	t=0	t=T			
		$S_T < K_A$	$K_A < S_T < K_B$	$K_B < S_T < K_C$	$K_C < S_T$
LONG CALL A	$-c_A$	0	$S_T - K_A$	$S_T - K_A$	$S_T - K_A$
2 SHORT CALLS B	$2c_B$	0	0	$-2(S_T - K_B)$	$-2(S_T - K_B)$
LONG CALL C	$-c_C$	0	0	0	$S_T - K_C$
TOTAL ($2c_B - c_A - c_C$)		0	$S_T - K_A > 0$	$-S_T + (2K_B - K_A)$ $= K_C - S_T > 0$	0



$$\pi_{\text{BUTTERFLY CALL SPREAD}} = \pi_{\text{LONG CALL A}} + 2\pi_{\text{SHORT CALL B}} + \pi_{\text{LONG CALL C}}$$

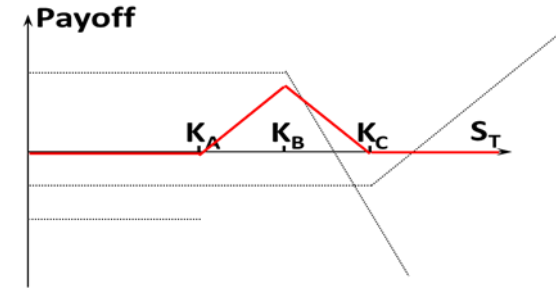
$$= \begin{cases} c_{2B,-A,-C} & \text{if } S_T < K_A < K_B < K_C \\ (S_T - K_A) + c_{2B,-A,-C} & \text{if } K_A < S_T < K_B < K_C \\ \begin{cases} (S_T - K_A) + 2(K_B - S_T) + c_{2B,-A,-C} \\ = -S_T + K_C + c_{2B,-A,-C} \text{ since } 2K_B = K_A + K_C \end{cases} & \text{if } K_A < K_B < S_T < K_C \\ \begin{cases} (S_T - K_A) + 2(K_B - S_T) + (S_T - K_C) + c_{2B,-A,-C} \\ = c_{2B,-A,-C} \end{cases} & \text{if } K_A < K_B < K_C < S_T \end{cases}$$



PROVE $c_A + c_C - 2c_B > 0$

THE PAYOFFS ON MATURITY OF THE BUTTERFLY SPREAD ARE

$$= \begin{cases} 0 & \text{if } S_T < K_A \\ (S_T - K_A) & \text{if } K_A < S_T < K_B \\ 2K_B - (S_T + K_A) = K_C - S_T & \text{if } K_B < S_T < K_C \\ 0 & \text{if } K_C < S_T \end{cases}$$



IN EACH CASE THESE PAYOFFS ARE NON NEGATIVE
HENCE, THE COST OF SETTING UP THIS STRATEGY
MUST BE POSITIVE SO THAT $c_A + c_C - 2c_B > 0$

BOX SPREAD

- A box spread is a combination of a bull call spread with strike prices K_A and K_B and a bear put spread with the same two strike prices if all options are European is a box spread.
- Thus, a box spread consists of:
- BULLISH CALL SPREAD ($K_A < K_B$)
- LONG CALL A (K_A, T, c_A) + SHORT CALL B (K_B, T, c_B)
- BEARISH PUT SPREAD
- SHORT PUT X (K_A, T, p_X) + LONG PUT Y (K_B, T, p_Y)



BOX SPREAD

	$t=0$	$t=T$		
	COST	$S_T < K_A$	$K_A < S_T < K_B$	$K_B < S_T$
LONG CALL A	c_A	0	$S_T - K_A$	$S_T - K_A$
SHORT CALL B	$-c_B$	0	0	$-(S_T - K_B)$
SHORT PUT X	$-p_X$	$-(K_A - S_T)$	0	0
LONG PUT Y	p_Y	$K_B - S_T$	$K_B - S_T$	0
TOTAL	$(c_A - c_B) + (p_Y - p_X)$	$K_B - K_A$	$K_B - K_A$	$K_B - K_A > 0$

BOX SPREAD CONTD...

- The payoff from a box spread is always $K_B - K_A$
The value of a box spread is therefore always the present value of this payoff or $(K_B - K_A)e^{-rT}$
- If it has a different value there is an arbitrage opportunity.



PAYOFF FROM BOX SPREAD ON MATURITY

STOCK VALUE	OPTION A	OPTION B	OPTION X	OPTION Y	TOTAL PAYOFF
$0 < S_T < K_A$	0	0	$-(K_A - S_T)$	$(K_B - S_T)$	$(K_B - K_A)$
$K_A < S_T < K_B$	$(S_T - K_A)$	0	0	$(K_B - S_T)$	$(K_B - K_A)$
$K_B < S_T$	$(S_T - K_A)$	$-(S_T - K_B)$	0	0	$(K_B - K_A)$

COST OF BOX SPREAD = $c_A - c_B - p_X + p_Y$ (outflow)

HENCE, WE MUST HAVE $c_A - c_B - p_X + p_Y = (K_B - K_A)e^{-rT}$



BOX SPREAD ARBITRAGE

- If the market price of the box spread is too low, it is profitable to buy the box. This involves buying a call with strike price K_A buying a put with strike price K_B , selling a call with strike price K_B , and selling a put with strike price K_A .
- If the market price of the box spread is too high, it is profitable to sell the box.
- It is important to realize that a box-spread arbitrage works with European options ONLY.



HORIZONTAL SPREAD/CALENDAR SPREAD

- It can be created by selling a call option with a certain strike price and buying a longer-maturity call option with the same strike price.
- The longer the maturity of an option, the more expensive it usually is.
- A calendar spread therefore usually requires an initial investment.



CALENDAR SPREAD: PROFIT PROFILE

- Profit diagrams for calendar spreads are usually produced so that they show the profit when the short-maturity option expires on the assumption that the long-maturity option is sold at that time.
- The profit pattern is similar to the profit from the butterfly spread.
- The investor makes a profit if the stock price at the expiration of the short-maturity option is close to the strike price of the short-maturity option. However, a loss is incurred when the stock price is significantly above or significantly below this strike price.



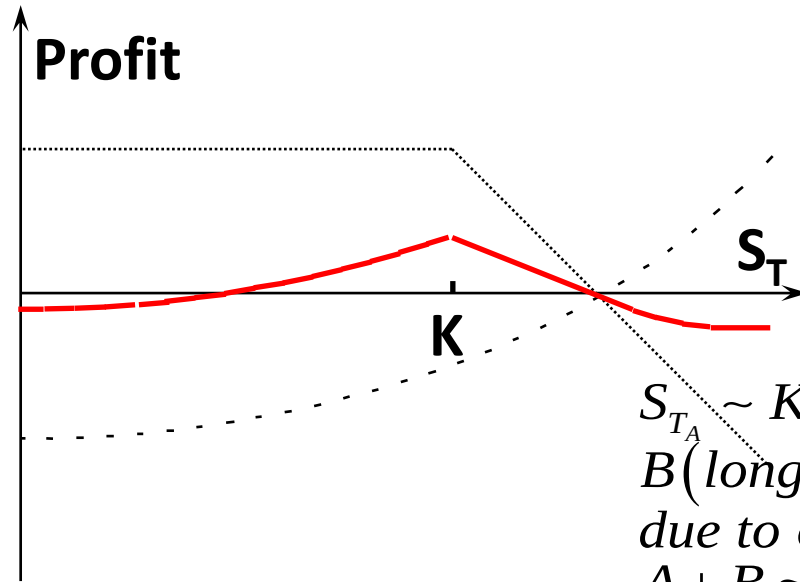
HORIZONTAL CALL SPREADS (CALENDAR CALL SPREADS):PROFILE

$$S_{T_A} \gg K$$

$$A(\text{short}) := -(S_{T_A} - K)$$

$$B(\text{long}) := (S_{T_A} - K)$$

$$A + B \approx 0$$



$$S_{T_A} \ll K$$

$$A(\text{short}) := 0$$

$$B(\text{long}) := 0$$

$$A + B \approx 0$$

$$S_{T_A} \sim K; A(\text{short}) := 0$$

$$B(\text{long}) : \text{Significant + value due to option's time value}$$

$$A + B \approx \text{Significant + value}$$

CALENDER SPREAD: PROFIT PROFILE EXPLANATION

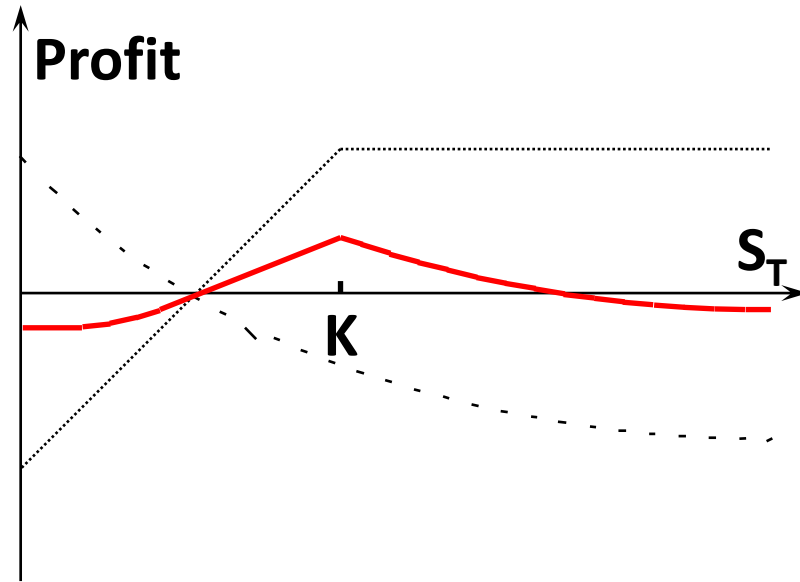
- if the stock price is very low when the short-maturity option expires, the short-maturity option is worthless and the value of the long-maturity option is close to zero. The investor therefore incurs a loss that is close to the cost of setting up the spread initially.



- Consider next what happens if the stock price, S_T , is very high when the short-maturity option expires. The short-maturity option costs the investor $S_T - K$, and the long maturity option is worth close to $S_T - K$, where K is the strike price of the options. Again, the investor makes a net loss that is close to the cost of setting up the spread initially.
- If S_T is close to K , the short-maturity option costs the investor either a small amount or nothing at all. However, the long-maturity option is still quite valuable. In this case a significant net profit is made.



HORIZONTAL PUT SPREADS (CALENDAR PUT SPREADS):PROFILE



EXAMPLE 1

- **“A box spread comprises four options. Two can be combined to create a long forward position and two can be combined to create a short forward position.” Explain this statement.**



- **Bull Call Spread:** $[A(K_A, T, c_A) - B(K_B, T, c_B)] +$
- **Bear Put Spread:** $[-X(K_A, T, p_A) + Y(K_B, T, p_B)]$
- **Long Forward:** $[A(K_A, T, c_A) - X(K_A, T, p_A)]$
- **Short Forward:** $-[B(K_B, T, c_B) - Y(K_B, T, p_B)]$

LONG CALL +SHORT PUT

	$t=0$	$t=T$	
		$S_T < K$	$S_T > K$
LONG CALL	$-c$	0	$S_T - K$
SHORT PUT	$+p$	$-(K - S_T)$	0
TOTAL	$-(c - p)$	$S_T - K$	$S_T - K$